

## FOREWORD

This special volume of *Theory and Applications of Categories* is dedicated to the memory of F. William (Bill) Lawvere (1937–2023), in appreciation of his profound and lasting contributions to category theory and, in a broad sense, logic and geometry.

Ever since the completion of his Ph.D. thesis in 1963, Bill’s visionary work deeply influenced major developments in those mathematical fields. In fact, it reached into subjects far beyond them, from philosophy to physics. His passionate approach to mathematics extended into many aspects of public life, from education to politics. People who had the privilege to collaborate with Bill, or to just casually interact with him, benefitted from his great generosity in sharing his insights.

Bill’s legacy in mathematics and beyond rests primarily on having made several fundamental concepts explicit—not the least of which were the notions of algebraic theory, elementary topos (with Myles Tierney), and (hyper)doctrine. Bill’s ability to see ideas and concepts through the lens of adjunction was extraordinary. His mathematical interpretation of Hegel’s *Aufhebung* was only one of many stellar examples in this regard.

For a full appreciation of Bill’s life and work, we recommend Anders Kock’s excellent article: <https://ems.press/journals/mag/articles/11170869>. This and many other resources may also be found in *The Lawvere Archives* at: <https://lawverearchives.com>. Below we include a list of Bill’s publications that is as complete as we were able to make it; we acknowledge *The Lawvere Archives* and the github page of Matt Earnshaw.

We thank the editors of *Theory and Applications of Categories* for the opportunity to prepare this volume, and Geoff Cruttwell, the Managing Editor, for his guidance. The articles have been carefully refereed according to the usual *TAC* standards. We are confident that they fall within the realm of Bill’s wide interests and thank the authors and the referees for their contributions.

The Guest Editors:

*Matías Menni, Bob Rosebrugh, Jiří Rosický and Walter Tholen*

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